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Analytical study of excitation and measurement of fluid-solid interface waves

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[1] Analytical analyses are performed to investigate the pressure and particle velocity responses in fluid/solid half spaces subject to impulsive sources. This study has significance for underwater seabed characterization efforts that use interface waves. Results show that pressure (e.g., measured by hydrophones) in the fluid is more sensitive for interface wave sensing than particle velocity (e.g., measured by geophones) at the fluid-solid interface. Both types of impulsive point wave sources, explosive action in fluid and mechanical action on solid, are investigated with regard to the excitability of interface waves. Analyses show that a mechanical load applied normal to the interface generates higher amplitude interface waves, relative to the acoustic wave amplitude, than an explosive load in the fluid. The effect of explosive source height is also investigated. Results show that Scholte wave amplitude is affected by the explosive source height, and decays quickly with increasing height. However, explosive source height has little effect on generated leaky Rayleigh wave amplitude.

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1. Introduction

[2] Two types of interface waves can exist in a fluid-elastic solid system: the Scholte wave and the leaky Rayleigh wave. Extensive studies on the behavior of fluid-solid interface waves have been reported over the past 40 years. A comprehensive study has been given by *Viktorov* [1967], where leaky Rayleigh waves at the interfaces between a solid halfspace and a fluid layer, and between solid halfspace and a fluid half-space, were investigated in great detail. Many practical studies of interface wave behavior have also been reported, for example *Chamuel and Brooke* [1988] studied the effect of rough and periodic solid surfaces on Scholte wave transmission characteristics, and *Adler and Nagy* [1994] studied the effect of fluid-filled porous solids on the velocity and attenuation of interface waves. More recently, the character and existence of leaky surface waves and Scholte waves were investigated experimentally [*Glorieux and Van de Rostyne*, 2001].

[3] Leaky Rayleigh waves exist only for stiff solid - light fluid (e.g., water) cases, where the shear wave velocity of the solid c_S is larger than the acoustic wave velocity in the fluid c_F. Scholte waves exist for any combination of fluids and solids. The fraction of Scholte wave energy that travels in the fluid and solid, respectively, depends on their relative densities and wave velocities. For the stiff solid – light fluid case, such as underwater seabed rock, more energy will be localized in the fluid than in the solid. On the other hand for a compliant solid with lower modulus and density, such as seafloor sediment, more energy will be localized in the solid; in this case Scholte waves have deeper penetration in the solid.

[4] Fluid-solid interface waves can be applied to characterize physical properties of the solid along which the waves travel, giving rise to geophysical testing application. For the stiff solid/fluid case, leaky Rayleigh waves (or general Rayleigh waves) can be used to characterize properties of seabed rock. Conventional Rayleigh wave dispersion measurement techniques, such as SASW (Spectral Analysis of Surface Waves) [*Nazarian*, 1984] and MASW (Multichannel Analysis of Surface Waves) [*Park et al.*, 1999] methods, have also been applied to measure shear wave velocities of underwater solids. For the compliant solid/fluid case, Scholte waves can be used. For example, Scholte waves have been successfully applied to characterize marine sediment [*Ritzwoller and Levshin*, 2002; *Bohlen et al.*, 2004] and underwater geotechnical sites [*Luke and Stokoe*, 1998].

[5] Both types of interface waves can be excited by impulsive events: either transient loads applied to the surface of the solid at the interface (e.g., a mechanical vibrator or free-falling projectile) [*Stoll et al.*, 1996; *Ohta et al.*, 2002] or an explosive source applied in the fluid (e.g., an air gun shot) [*Ritzwoller and Levshin*, 2002; *Bohlen et al.*, 2004]. However, an explosive source generates strong compressional waves in the fluid and solid that tend to contaminate interface wave measurement. In addition, the transient pressure characteristics are usually not available for explosive sources [*Stoll et al.*, 1996]. The resulting interface waves are monitored either by measuring particle velocities on the solid at the interface using geophones or pressure in the fluid using hydrophones. Analytical solutions for the wave field in a fluid-solid configuration resulting from explosive line and point sources, respectively, in the fluid have been given by *de Hoop and van der Hijden* [1983, 1984]. The wave field response owing to transient point loading of the solid at the interface is also of practical interest to geophysical work where a vibrator or falling projectile is applied to generate the waves. Recently *Zhu et al.* [2004] derived the exact solution of the complete wave fields in the solid and fluid owing to a point load that is applied normally to the solid at the interface, and used this solution to guide application of leaky Rayleigh waves for non-destructive characterization of a solid material (concrete) [*Zhu and Popovics*, 2005].

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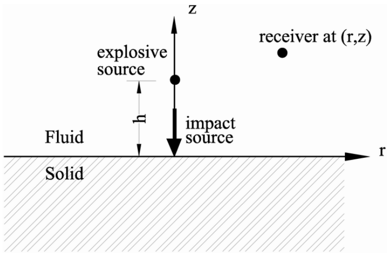

Figure 1. Assumed coordinates and location of sources and receivers in the fluid-solid half-space system.

[6] A study of interface wave excitation and sensing has practical value for geophysical studies that characterize underwater solids using these waves. In this paper, the effectiveness of interface wave excitation and detection is studied using analytical wave field models considering two cases: stiff solid/water and compliant solid/water. Both types of impulsive wave sources are considered: for explosive sources in fluids the model of *de Hoop and van der Hijden* [1984] is applied and that of *Zhu et al.* [2004] for mechanical loads at the interface.

2. Interface Waves Responses

[7] The complete solution for Lamb's problem for a fluid/solid half space system has been given by *Zhu et al.* [2004], who gave expression of pressure $P(t)$ and vertical displacement $u_z(t)$ in the fluid owing to a normal point load applied on the fluid-solid interface. A cylindrical system of coordinates are employed where the *z*-axis is normal to the interface between the two media and passes through the source, and the origin is at the fluid-solid interface; the r-axis lies along the interface with the origin at the z-axis intersection. Figure 1 shows the coordinates and relative position of sources and receiver. When applying their model for a fluid/stiff solid configuration (an impact force on the interface), *Zhu et al.* [2004] have shown that the acoustic wave contribution is negligible, therefore the transient signals are dominated by Scholte wave and leaky R-wave contributions. A further implication is that the pressure response in the water is similar to the velocity measured at the interface, except there is a phase reversal in the Scholte wave pulse [*Zhu*, 2005].

[8] The particle trajectories of leaky Rayleigh waves and Scholte waves in the fluid are elliptical. According to *Viktorov* [1967], the amplitude ratio between vertical (z) and horizontal (r) components of displacement is

$$u_z/u_r = \left|\sqrt{1 - c^2/c_F^2}\right|, \qquad (1)$$

where c is the phase velocity of either the leaky Rayleigh wave c_{LR} or the Scholte wave c_{sch}. Because Scholte wave velocity c_{sch} is always less than c_F, $u_z/u_r < 1$ always. Therefore the major axis of the trajectory ellipse is in the propagation direction. For fluid/stiff solid configurations, c_{sch} is very close to c_F, therefore u_r will be much larger than u_z, and the trajectory ellipse will be very flat. For leaky Rayleigh waves, the major axis of the trajectory ellipse is in the vertical direction when $c_{LR} > \sqrt{2}c_F$. The same relation applies to particle velocity (v) too.

[9] Fluid pressure is related to the magnitude of fluid velocity by

$$P(t) = \rho v^2(t), \qquad (2)$$

where $v^2(t) = v_z^2(t) + v_r^2(t)$, considering contributions from all directions. Because the horizontal component dominates the Scholte wave response, it is inferred that the pressure response of Scholte waves will be stronger than the vertical (z) component response of particle velocity. To illustrate this point, P(t) and v_z(t) owing to a mechanical impact force $f(t) = \sin^2(\pi t/T)$ with input force magnitude of 1.0 kN were computed nearby the source (r = 1.5 m) for the stiff solid case. The near-surface (z = 0.005 m) peak pressures of the leaky Rayleigh wave and Scholte wave are 0.23 kPa and 1.09 kPa respectively, both of which can be readily measured by hydrophones with sensitivity above −200 dB (0.1 V/kPa). The corresponding fluid velocities back-calculated from equation (2) are 0.48 m/s and 1.044 m/s, assuming density of water is 1000 kg/m^3. However, for the same loading condition, the peak v_z at the interface (z = 0) induced by leaky Rayleigh waves and Scholte waves are only 0.105 mm/s and 0.19 mm/s respectively, which are difficult to detect using conventional geophones. Therefore we conclude that measuring fluid pressure with hydrophones is more effective than measuring interface particle velocity v_z with vertical-component geophones for interface waves generated by mechanical point load excitation.

3. Interface Waves Generated by Impulsive Sources

3.1. Scholte Waves at the Fluid/Compliant Solid Interface

[10] Scholte waves can be excited by an explosive source in the fluid or a normal point load on the solid at the interface. To investigate the excitability of Scholte waves by different types of sources, pressure responses for the water/compliant solid configuration are considered, where shear wave velocity in the solid c_S is lower than c_F. Because the Scholte wave velocity is lower than both c_S and c_F, the Scholte wave pulse arrival can be distinguished from acoustic waves in water within the time domain signal.

[11] Figure 2 shows pressure responses at r = 1.5 m, z = 0.05 m in water resulting from an impact point load and an

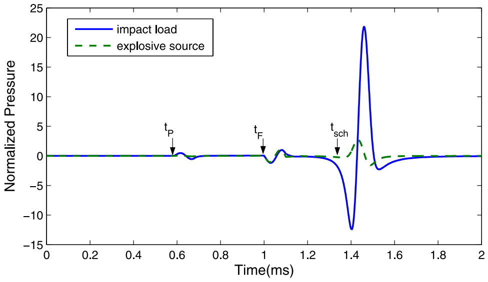

Figure 2. Excitability of Sholte waves by a normal point load at water/compliant solid interface and an explosive source in the water at $r = z = 0$ m. Pressures are normalized with respect to that of acoustic waves. Assumed material properties are for water ρ_1 = 1000 kg/m^3, c_F = 1500 m/s and for the compliant solid are ρ_2 = 1200 kg/m^3, c_P = 2700 m/s, Poisson's ratio ν = 0.31.

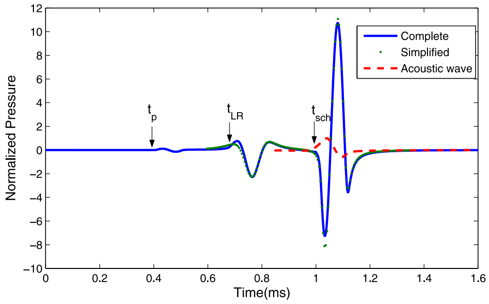

Figure 3. Excitability of interface waves at r = 1.5 m, z = 0.05 m in water by a mechanical impact point load on the water/stiff solid interface at $r = z = 0$ m. Pressures are normalized to the acoustic wave response that is obtained from the difference between the complete and simplified solutions. Assumed material properties are for water ρ_1 = 1000 kg/m^3, c_F = 1500 m/s and for the stiff solid ρ_2 = 2400 kg/m^3, c_P = 4000 m/s, Poisson's ration ν = 0.25.

explosive source. The explosive source is located in water, infinitely close to the interface at the origin. Because c_S is lower than c_F, leaky shear waves and leaky Rayleigh waves do not exist. Only leaky P waves, direct acoustic waves and Scholte waves exist, and are indicated by the three pulses shown in Figure 2. Pressure amplitudes are normalized with respect to that of the direct acoustic wave pulse. It is seen that the normalized Scholte wave amplitude excited by a normal point source is 8 times that of the explosive source, confirming the observation that a normal point load is more effective to generate Scholte waves than an explosive source [*Stoll et al.*, 1996]. A similar conclusion is obtained for leaky *P* waves. Reducing the relative amplitude of acoustic waves will improve measurement accuracy for interface waves, especially when it is difficult to differentiate interface waves from acoustic waves in the time domain because their arrival times are similar.

3.2. Leaky Rayleigh Waves at the Fluid/Stiff Solid Interface

[12] To investigate the excitability of leaky Rayleigh waves by a transient normal point load, a fluid/stiff solid case is studied. In this case, Scholte wave velocity c_{sch} = 1471 m/s is close to the fluid acoustic velocity in water c_F = 1500 m/s. Therefore, the Scholte wave response cannot be distinguished from the fluid acoustic waves in the time domain signal. However, a simplified solution of leaky Rayleigh and Scholte wave fields excited by a normal point load has been derived by *Zhu et al.* [2004]. By calculating residues from corresponding poles, the simplified solution contains contributions only from leaky Rayleigh and Scholte waves. Figure 3 shows the complete and simplified solutions of pressure at r = 1.5 m, z = 0.05 m in water. The largest pulse in Figure 3 coincides with the Scholte wave and acoustic wave arrival, and a smaller pulse at t = 0.6 ms coincides with the leaky Rayleigh wave arrival. Overall the complete and simplified solutions agree. A difference in two solutions nearby the Scholte wave arrival is seen, which is a result of the fact that the acoustic waves are not considered in the simplified solution. The acoustic wave response can thus be isolated and obtained as the difference between the complete and simplified solutions. In Figure 3, the responses are normalized with respect to peak amplitude of this acoustic wave response. By comparing the curves in Figure 3, we conclude that the response is dominated by the leaky Rayleigh wave and Scholte wave. The normalized amplitude of the leaky Rayeligh wave is about 2.3, and that of the Scholte waves is about 10.7.

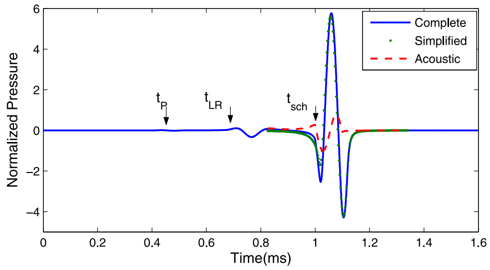

Figure 4. Excitability of interface waves at r = 1.5 m, z = 0.05 m in water by an explosive source for the water/stiff solid configuration. Pressures are normalized to the acoustic wave response obtained from the difference between the complete and simplified solutions. Assumed material properties are the same as those in Figure 3.

[13] Figure 4 shows the normalized pressure at the same position generated by an explosive load in the fluid at z = 0.005m. The complete solution is given by *de Hoop et al.* [1984], while the simplified solution is derived by the authors using the residue approach. As before, the acoustic wave response is obtained from the difference between the complete and simplified solutions, and shown in Figure 4 as a dashed line. In this case the response is dominated by the Scholte wave, which has normalized amplitude of 5.8, while the leaky Rayleigh wave amplitude is only 0.33.

[14] Considering the results from Figures 3 and 4, we conclude that the normal point load is more effective for excitation of leaky Rayleigh waves than the explosive source. With respect to the acoustic wave amplitude, the leaky Rayleigh wave excitability from the normal point load is about 7 times of that of an explosive source. Both sources are effective for excitation of Scholte waves.

3.3. Effect of Explosive Source Height on Interface Wave Excitability

[15] The excitability of interface waves generated by an explosive source depends on the height of the source above the interface. Figure 5 shows pressure responses generated

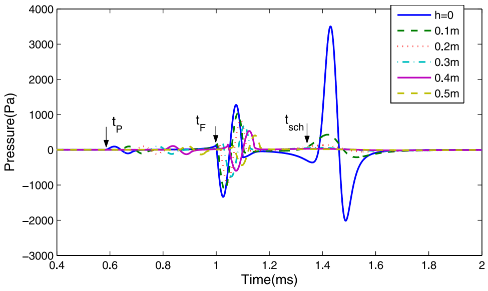

Figure 5. Pressure responses generated by an explosive source at six height positions h = 0 ∼ 0.5 m. The pressures were calculated at r = 1.5 m, z = 0.05 m in water for the water/compliant solid configuration. Assumed material properties are the same as those in Figure 2.

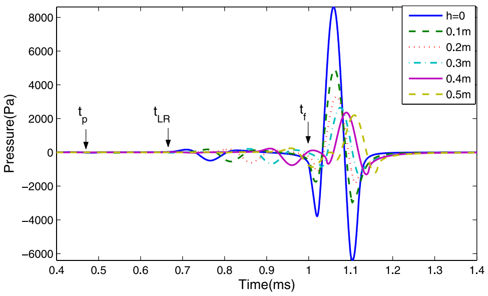

Figure 6. Pressure responses generated by an explosive source at six height positions h = 0 ∼ 0.5 m. The pressures were calculated at r = 2.5 m, z = 0.05 m in water for the water/stiff solid configuration. Assumed material properties are the same as those in Figure 3.

by an explosive source at six positions with height h = 0 ∼ 0.5 m. The pressures were calculated at r = 1.5 m, z = 0.05 m in water for the water-compliant solid configuration. As shown in the figure, arrival times of leaky P waves vary linearly with the source height *h*, while the amplitude remains relatively constant. The second and third pulses in the signals coincide with acoustic wave and Scholte wave arrivals, respectively. The decay pattern of Scholte wave amplitude with increasing source height indicates that Scholte waves dissipate quickly, on the order of h^{-2}, with source height. This observation agrees with *Ritzwoller and Levshin*'s [2002] results.

[16] The effect of source height on pressures in the water/stiff solid configuration is shown in Figure 6. Unlike Scholte waves, the amplitude of leaky Rayleigh waves are largely unaffected by source height. In fact, leaky Rayleigh waves do not decay with increasing source height, but rather show a slight increase in amplitude. Therefore, the source can be positioned at any height above the interface when an explosive source is used to generate leaky Rayleigh waves,. However, one must maintain proper source-receiver spacing, so that leaky Rayleigh waves can be separated from the acoustic waves in the time domain.

4. Conclusions

[17] Applying Green's functions derived by *Zhu et al.* [2004] and *de Hoop and van der Hijden* [1984], this analytical study investigates responses in fluid/solid half spaces subject to both types of impulsive sources. Particle velocity measured at the interface was compared to pressure measured in the fluid. Results show that pressures (e.g., hydrophones in the fluid) are more practical than surface velocity (e.g., vertical geophones at the surface) for interface wave sensing with respect to instrument sensitivity. By isolating the contributions of interface waves from acoustic and other waves, the effectiveness of the two types of impulsive point sources were investigated. With respect to the amplitude of acoustic waves generated, the impact point load applied normal to the interface is more effective than the explosive load in the fluid for exciting interface waves. The effect of explosive source height was also investigated. Analysis shows that excited Scholte wave amplitude decays quickly with increasing source height, whereas leaky Rayleigh wave amplitude is insensitive to the source height. The research results can be used as guidance for interface wave generation and measurement, in particular to characterize underwater structures.

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